Reinforcement Learning for the Game of Formal Mathematics

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Plan for today

What is formal mathematics?

Mathematics is (mostly) informal

The foundational crisis

The foundational crisis

Can we do any mathematics from axioms and inference rules?

C is getting smaller

Lean

Lean

Example: Natural numbers

A first theorem: `add_zero`

Let's prove step-by-step

```
theorem zero_add (n : Nat) : add zero n = n := by
```

Proving `zero_add`, continued

Goal 2 (inductive step):

```
n : Nat
ih : add zero n = n
⊢ add zero (succ n) = succ n
```

Building up: more lemmas

Tactics: the vocabulary

The game of formal theorem proving

The MDP: State and Action

The MDP: Transition and Reward

The game

Chess / Go	Theorem proving

Key challenges

Two initial learning signals

The general recipe (preview)

Part I: The classical era (2016–2020)

The "modern classical" era

Holophrasm: a neural Automated Theorem Prover for higher-order logic

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Holophrasm: search and results

Figure 1: Examples of an unsolved, the top, and a proposition, bottom. The Metric Prover is unable to solve any of the nodes, and is forced to backtrack after each unsuccessful attempt at a proof step.

Reinforcement Learning of Theorem Proving

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Learning to Prove Theorems via Interacting with Proof Assistants

Kaiyu Yang [1] Jia Deng [1]

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Generative Language Modeling for Automated Theorem Proving

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Summary of Part I

	Policy	Value	Search	Training signal

PART III: THE ROAD TO IMO

miniF2F: a shared benchmark

Why tactics make this much difference

Formal Mathematics Statement Curriculum Learning

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Strategy 1: synthetic data

Strategy 2: manual formalization

Expert iteration on all statements

Combine everything: mathlib + synthetic inequalities + manually formalized statements. Run expert iteration:

Results on miniF2F

HyperTree Proof Search for Neural Theorem Proving

Guillaume Lample [1] **Marie-Anne Lachaux** [1] **Thibaut Lavril** [1] **Xavier Martinet** [1]
Amaury Hayat[2] **Gabriel Ebner**[3] **Aurélien Rodriguez**[1] **Timothée Lacroix**[1,4]

HTPS: results

Also introduces **online training**: trainer and prover run concurrently, model weights updated continuously (vs. discrete expert iteration rounds).

PART IV: BEYOND IMO

BREAKING THE DATA WALL

The data wall

DeepSeek-Prover: Advancing Theorem Proving in LLMs through Large-Scale Synthetic Data

Huajian Xin[1,*] Daya Guo[1,*] Zhihong Shao[1,*] Z.Z. Ren[1] Qihao Zhu[1] Bo Liu[2] Chong Ruan[2] Wenda Li[3] Xiaodan Liang[1,4]
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Step 1: Autoformalization

Use an LLM to translate natural language problems into Lean 4 statements:

Hypothesis rejection

DeepSeek-Prover V1: results

Table 1 | Comparing with state-of-the-arts on the miniF2F dataset.

Method	Model size	Generation times	miniF2F-valid	miniF2F-test
Best-first search Method				
GPT-f (Polu et al., 2020)	0.7 B	2 × 512	36.6%	36.6%
PACT (Han et al., 2021)	0.7 B	2 × 512	37.7%	35.2%
Proof Artifact Co-training (Lample et al., 2022)	600M	2 × 512	39.3%	36.3%
	1.4B	2 × 512	41.0%	38.5%
	2.8B	2 × 512		37.1%
Hypertree Proof Search (Lample et al., 2022)	600M	64 × 5000 simulations	58.6%	-
	1.4B	64 × 5000 simulations		41.0%
Whole-proof generation Method				
Baldur (First et al., 2023)	7B	64	55.3%	50.0%
GPT-f-cube-300	7B	64	41.2%	39.8%
Draft-Sketch-Prove (Jiang et al., 2022)	7B	1 (pass@1)	42.6%	39.8%
		8 (pass@8)	-	50.0%
DeepSeek-Prover	7B	128	-	46.3%
		4096	-	50.0%
		8192	-	52.0%

DeepSeek-Prover-V1: Advancing Formal Mathematical Reasoning via Reinforcement Learning for Subgoal Decomposition

Huajian Xin, Z.Z. Ren, Junxiao Song, Zhihong Shao, Wanjia Zhao, Haocheng Wang, Bo Liu, Liyue Zhang, Xuan Lu, Qiushi Du, Wenjun Gao, Qihao Zhu, Dejian Yang, Zhibin Gou, Z.F. Wu, Fuli Luo, Chong Ruan
DeepSeek-AI

https://github.com/deepseek-ai/DeepSeek-Prover-V1

Subgoal decomposition

The reasoning LLM produces a Lean skeleton with `sorry` placeholders. A specialized 7B prover fills each one:

Subgoal decomposition → much longer proofs

IMO 1992 Problem 1 — a ∼280-line proof:

DeepSeek-Prover V2: results

Method	Model size	Sample budget	miniF2F-test
Non-CoT methods			
DeepSeek-Prover-V1.5-RL + RMaxTS (Dong and Ma, 2025)	7B	32 × 400	63.5%
DeepSeek-Prover-V1.5-RL + RMaxTS (Dong and Ma, 2025)	7B	256 × 32 × 400	65.6%
BFS-Prover (Xin et al., 2025)	7B	2048 × 2 × 600	70.8%
	7B	2048 × 2 × 1024	72.1%
CoT methods			
Leanabell-Prover-V2 (w/ CoT) (Ji et al., 2025)	7B	128	61.1%
Leanabell-Prover-V2 (w/ RL) (Ji et al., 2025)	7B	128	63.9%
Kimina-Prover-Preview-Distill (Wang et al., 2025)	1.5B	1	39.3%
		32	63.9%
	7B	1	52.5%
		32	68.0%
	72B	1	65.5%
Kimina-Prover-Preview (Wang et al., 2025)		32	80.7%
	72B	1	63.9%
		32	82.0%
DeepSeek-Prover-V2 (ours)	7B	1	58.6% / 62.9%
		32	75.6% / 77.0%
		8192	82.0% / 83.2%
	671B	1	61.9% / 64.7%
		32	78.6% / 79.9%
		8192	86.6% / 86.9%
DeepSeek-Prover-V2 (CoT)	7B	1	63.5% / 64.3%
		32	79.9% / 81.6%
		8192	86.9% / 87.3%
	671B	1	82.4% / 82.0%
		32	90.2% / 89.8%
		8192	90.2% / 91.0%

Article

Solving olympiad geometry without human demonstrations

https://doi.org/10.1038/s41586-023-06747-5 Trieu H. Trinh[1,2,*], Yuhuai Wu[1], Quoc V. Le[1], He He[2,3] & Thang Luong[1,*]

Synthetic data generation for geometry

Synthetic auxiliary constructions

AlphaGeometry: results

Article

Table 1 | Main results on our IMO-AG-30 test benchmark

Method	Training data and setup	Solved (out of 30)
Automatic Algebra	Human proofs*, elimination rules and Wu's method	4
	Gröbner basis*	4
Search Symbolic Deduction	Human proofs*	7
	Full angle method*	8
	Full deductive database*	9
	DD (Gaussian elimination)*	14
	DD+AR*	15
	DD+AR (human-like traceback)*	25
	DD+AR (ours)	14
	DD+AR (ours) + synthetic data	21
	DD+AR+SC (ours)	21
	DD+AR+SC (ours) + synthetic data	25

*Human proofs, human-designed rules and Wu's method are from the 2000–2022 IMO competitors. We manually translated the inputs to the domain-specific languages of each symbolic engine. The outputs are then converted back to human readable proofs for verification. DD, AR and SC stand for deductive database, algebraic reasoning and search for auxiliary points, respectively. Synthetic data refers to the 100 million examples we generated.

A more general proof

On IMO 2004 P1, AlphaGeometry found a proof that was **more general** than the original problem:

...but sometimes more verbose

Given: ABC orthocenter
H, D ∈ BC, E ∈ AC
AD ⊥ BC, BE ⊥ AC
Prove: DE ⊥ DH
Proof:
Let AD ∩ BE = H
Let AD ∩ BE = H
Let AD ∩ BE = H