Advanced Policy Optimization

Sham Kakade and Kianté Brantley

CS 2824: Foundations of Reinforcement Learning

Given two policies π and π', what is the performance difference: $V^{\pi}(s_0) - V^{\pi'}(s_0) = ??$

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(Diff in performances $\Leftarrow \Rightarrow$ Diff in policies?)

Setting and Notation

Discounted infinite horizon MDP:

$$\mathscr{M} = \{S, A, \gamma, r, P\}$$

State visitation: $d^{\pi}_{s_0}(s) = (1-\gamma)\sum_{h=0}^{\infty} \gamma^h \mathbb{P}^{\pi}_h(s; s_0)$

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Recall PI:

$$\arg\max_a Q^{\pi}(s,a) = \arg\max_a A^{\pi}(s,a),$$

i.e., Policy-improve step seeks the action that has the **largest adv**

PDL:

Given two policies $\pi : S \mapsto \Delta(A),\ \pi' : S \mapsto \Delta(A)$, recall $V^{\pi}(s_0) = \mathbb{E}\left[\sum_{h=0}^{\infty} \gamma^h r(s_h, a_h) \,|\, \pi\right]$

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Performance Difference Lemma (PDL):

$$V^{\pi}(s_0) - V^{\pi'}(s_0) = \frac{1}{1-\gamma}\mathbb{E}_{s\sim d^{\pi}_{s_0}}\left[\mathbb{E}_{a\sim\pi(\cdot|s)}Q^{\pi'}(s,a) - V^{\pi'}(s)\right]$$

$$:= \frac{1}{1-\gamma}\mathbb{E}_{s\sim d^{\pi}_{s_0}}\left[\mathbb{E}_{a\sim\pi(\cdot|s)}A^{\pi'}(s,a)\right]$$

PDL Explanation

$$V^{\pi}(s_0) - V^{\pi'}(s_0) = \frac{1}{1-\gamma}\mathbb{E}_{s\sim d^{\pi}_{s_0}}\left[\mathbb{E}_{a\sim\pi(\cdot|s)}A^{\pi'}(s,a)\right]$$

PDL Proof

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Proof of Sketch (see reading material for detailed steps)

PDL Proof

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$$V^{\pi}(s_0) - V^{\pi'}(s_0)$$

$$= V^{\pi}(s_0) - \mathbb{E}_{a_0\sim\pi(\cdot|s_0)}\left[r(s_0,a_0) + \gamma\mathbb{E}_{s'\sim P(s_0,a_0)}V^{\pi'}(s')\right] + \mathbb{E}_{a_0\sim\pi(\cdot|s_0)}\left[r(s_0,a_0) + \gamma\mathbb{E}_{s'\sim P(s_0,a_0)}V^{\pi'}(s')\right] - V^{\pi'}(s_0)$$

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$$= \gamma\mathbb{E}_{a_0\sim\pi(\cdot|s_0)}\mathbb{E}_{s_1\sim P(s_0,a_0)}\left[V^{\pi}(s_1) - V^{\pi'}(s_1)\right] + \mathbb{E}_{a_0\sim\pi(\cdot|s_0)}\left[r(s_0,a_0) + \gamma\mathbb{E}_{s'\sim P(s_0,a_0)}V^{\pi'}(s')\right] - V^{\pi'}(s_0)$$

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Outline for Today:

1. Trust Region Policy Optimization

2. Proximal Policy Optimization (PPO)

3. Group Relative Policy Optimization

Trust Region Policy Optimization

Want to manipulate $V^{\pi_\theta}(s_0)$ **into an objective** that we can estimate from sampled data

$$V^{\pi_\theta}(s_0) - V^{\pi_{\theta_t}}(s_0) = \frac{1}{1-\gamma}\mathbb{E}_{s\sim d_{s_0}^{\pi_\theta}}\left[\mathbb{E}_{a\sim\pi_\theta(\cdot|s)}A^{\pi_{\theta_t}}(s,a)\right]$$

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We know the first order Taylor expansion of $L_{\theta_t}(\theta)$

$$L_{\theta_t}(\theta) \approx L_{\theta_t}(\theta_t) + \nabla_\theta L_{\theta_t}(\theta_t)^\top(\theta-\theta_t)$$

Recap on Natural Policy Gradient:

At iteration t:

$$\max_{\pi_\theta} \mathbb{E}_{s\sim d_\mu^{\pi_{\theta_t}}} \left[\mathbb{E}_{a\sim\pi_\theta(s)} A^{\pi_{\theta_t}}(s,a) \right]$$

$$\text{s.t., } KL\left(\rho_{\pi_{\theta_t}} | \rho_{\pi_\theta}\right) \leq \delta$$

Intuition: maximize local adv subject to being incremental (in KL);

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$$\max_\theta \nabla_\theta J(\pi_{\theta_t})^\top (\theta - \theta_t)$$
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$$\text{s.t. } (\theta-\theta_t)^\top F_{\theta_t}(\theta-\theta_t) \leq \delta$$

$$\theta_{t+1} = \theta_t + \eta F_{\theta_t}^{-1} \nabla_\theta J(\pi_{\theta_t})$$

NPG

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NPG

$$F_{\theta_t} := \mathbb{E}_{s,a\sim d_\mu^{\pi_{\theta_t}}}\left[\nabla_\theta \ln \pi_{\theta_t}(a|s)\left(\nabla_\theta \ln \pi_{\theta_t}(a|s)\right)^\top\right] \in \mathbb{R}^{dim_\theta \times dim_\theta}$$

Policy Gradient (e.g., REINFORCE) can unstable and slow

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Natural Policy gradient is computational expensive

Even compute fisher information matrix is slow

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An extension of NPG (even faster in practice)

Given a current policy π^t, we perform policy update to π^{t+1}

Proximal Policy Optimization (PPO)

$$\max_{\theta} \; \mathbb{E}_{s\sim d_\mu^{\pi_{\theta_t}}}\left[\mathbb{E}_{a\sim\pi_\theta(\cdot|s)} A^{\pi_{\theta_t}}(s,a)\right] - \lambda \underbrace{\mathbb{E}_{s\sim d_\mu^{\pi_t}}\left[\mathrm{KL}\Big(\pi_{\theta_t}(a \mid s) \,\|\, \pi_\theta(a \mid s)\Big)\right]}_{\text{regularization}}$$

Proximal Policy Optimization (PPO)

Construct a **batch Supervised Learning** style objective using $\mathscr{D} = \{s, a, A^{\pi_{\theta_t}}(s, a)\}$

$$\max_{\theta} \ell(\theta) = \max_{\theta} \mathbb{E}_{s \sim d^{\pi_{\theta_t}}} \mathbb{E}_{a \sim \pi_{\theta}(\cdot|s)} \cdot A^{\pi_{\theta_t}}(s, a)$$

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IW trick $$\rightarrow \mathbb{E}_{s\sim d^{\pi_{\theta_t}}} \mathbb{E}_{a\sim\pi_{\theta_t}(\cdot|s)} \frac{\pi_\theta(a \mid s)}{\pi_{\theta_t}(a \mid s)} \cdot A^{\pi_{\theta_t}}(s, a)$$

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$$\approx \frac{1}{N} \sum_{s,a} \frac{\pi_\theta(a \mid s)}{\pi_{\theta_t}(a \mid s)} \cdot A^{\pi_{\theta_t}}(s, a)$$

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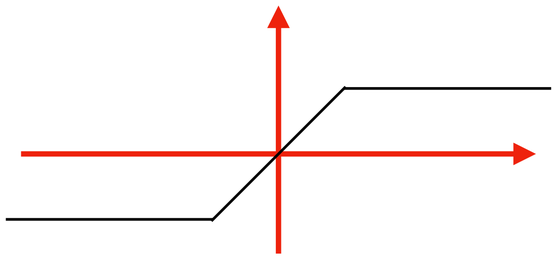

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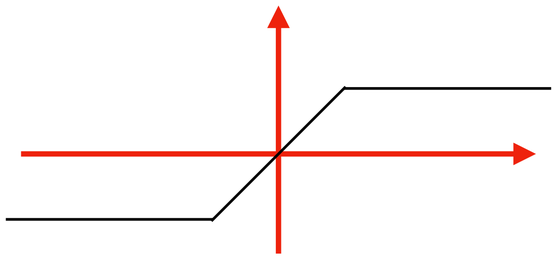
Stop updating $\pi_\theta(a \mid s)$ if it is too different from $\pi_{\theta_t}(a \mid s)$



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Proximal Policy Optimization (PPO)

Trick 2, take the min of the clipped and uncipped (original) obj

$$\hat{\ell}_{final}(\theta) = \sum_{s,a} \min \left\{ \frac{\pi_\theta(a \mid s)}{\pi_{\theta_t}(a \mid s)} \cdot A^{\pi_{\theta_t}}(s,a), \quad \text{clip}\left(\frac{\pi_\theta(a \mid s)}{\pi_{\theta_t}(a \mid s)}, 1-\epsilon, 1+\epsilon \right) \cdot A^{\pi_{\theta_t}}(s,a) \right\}$$

Original obj

clipped obj which ensures no abrupt change in action probabilities

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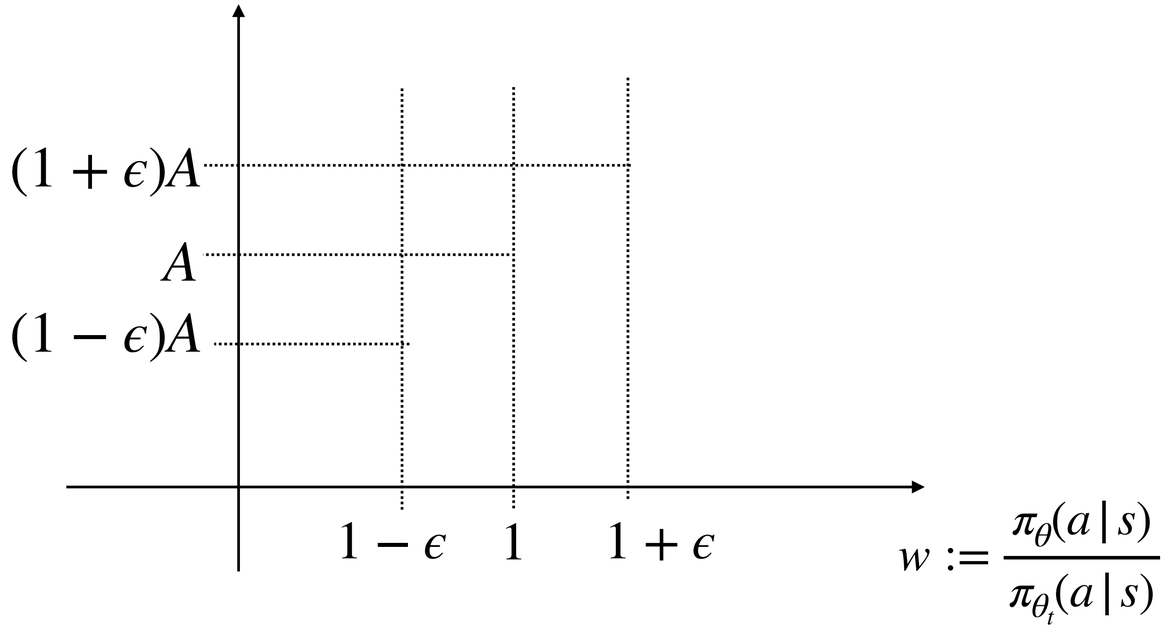
When $A^{\pi_{\theta_t}}(s,a) > 0$



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$$\hat{\ell}_{final}(\theta) = \sum_{s,a} \min \left\{ \frac{\pi_\theta(a \mid s)}{\pi_{\theta_t}(a \mid s)} \cdot A^{\pi_{\theta_t}}(s,a), \quad \text{clip}\left(\frac{\pi_\theta(a \mid s)}{\pi_{\theta_t}(a \mid s)}, 1-\epsilon, 1+\epsilon \right) \cdot A^{\pi_{\theta_t}}(s,a) \right\}$$

Just consider one term inside the summation:

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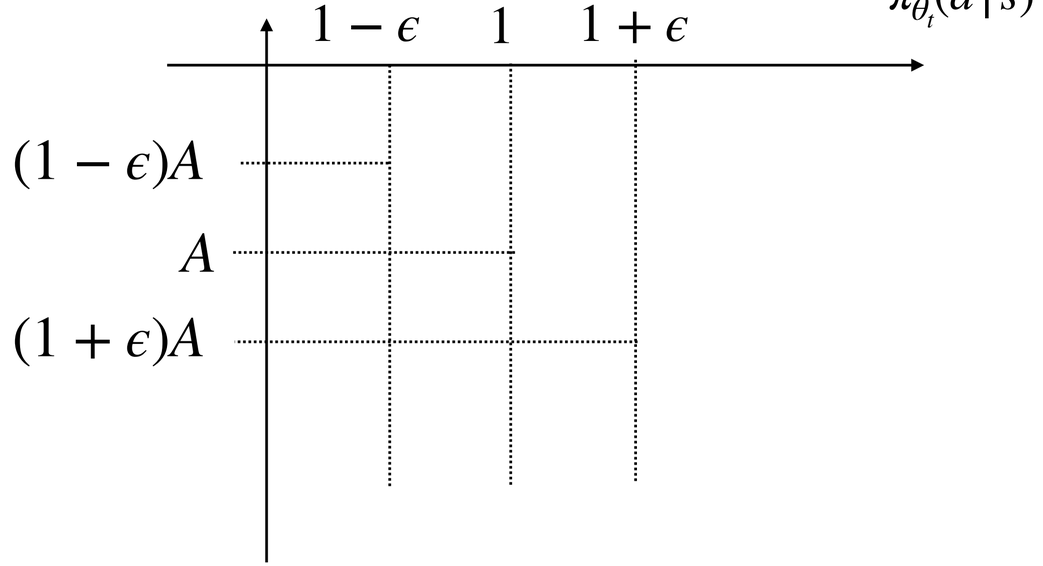
When $A^{\pi_{\theta_t}}(s,a) < 0$

$$w := \frac{\pi_\theta(a \mid s)}{\pi_{\theta_t}(a \mid s)}$$



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clipped obj which ensures no abrupt change in action probabilities

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We compute $\theta_{t+1} \approx \arg\max_\theta \hat{\ell}_{final}(\theta)$, via performing a few epoches of minbatch SG ascent (or Adam/Adagrad) on $\hat{\ell}_{final}$

Proximal Policy Optimization (PPO)

Initialize θ_0 for the policy

For $t = 0 \rightarrow T$:

Run π_{θ_t} to collect multiple trajectories, and form the dataset $\{s, a, A^{\pi_{\theta_t}}(s, a)\}$

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Perform a few steps of mini-batch gradient updates on $\hat{\ell}_{final}(\theta)$ to get θ_{t+1}

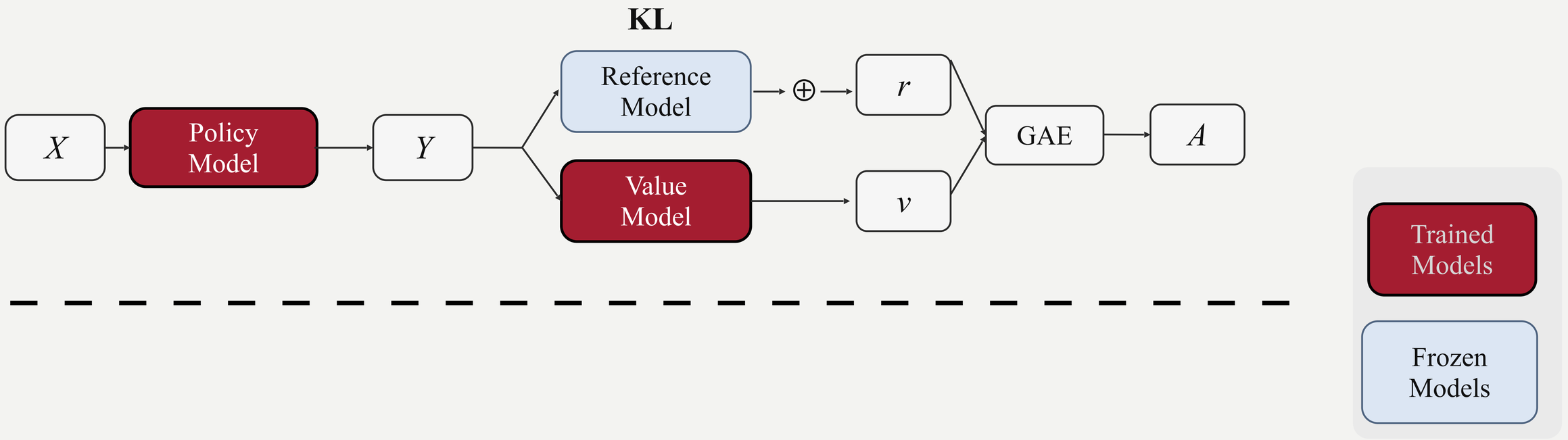
Outline for Today:

1. Trust Region Policy Optimization

2. Proximal Policy Optimization (PPO)

3. Group Relative Policy Optimization

PPO


KL
X
Policy Model
Y
Reference Model
Value Model
r
v
GAE
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Trained Models
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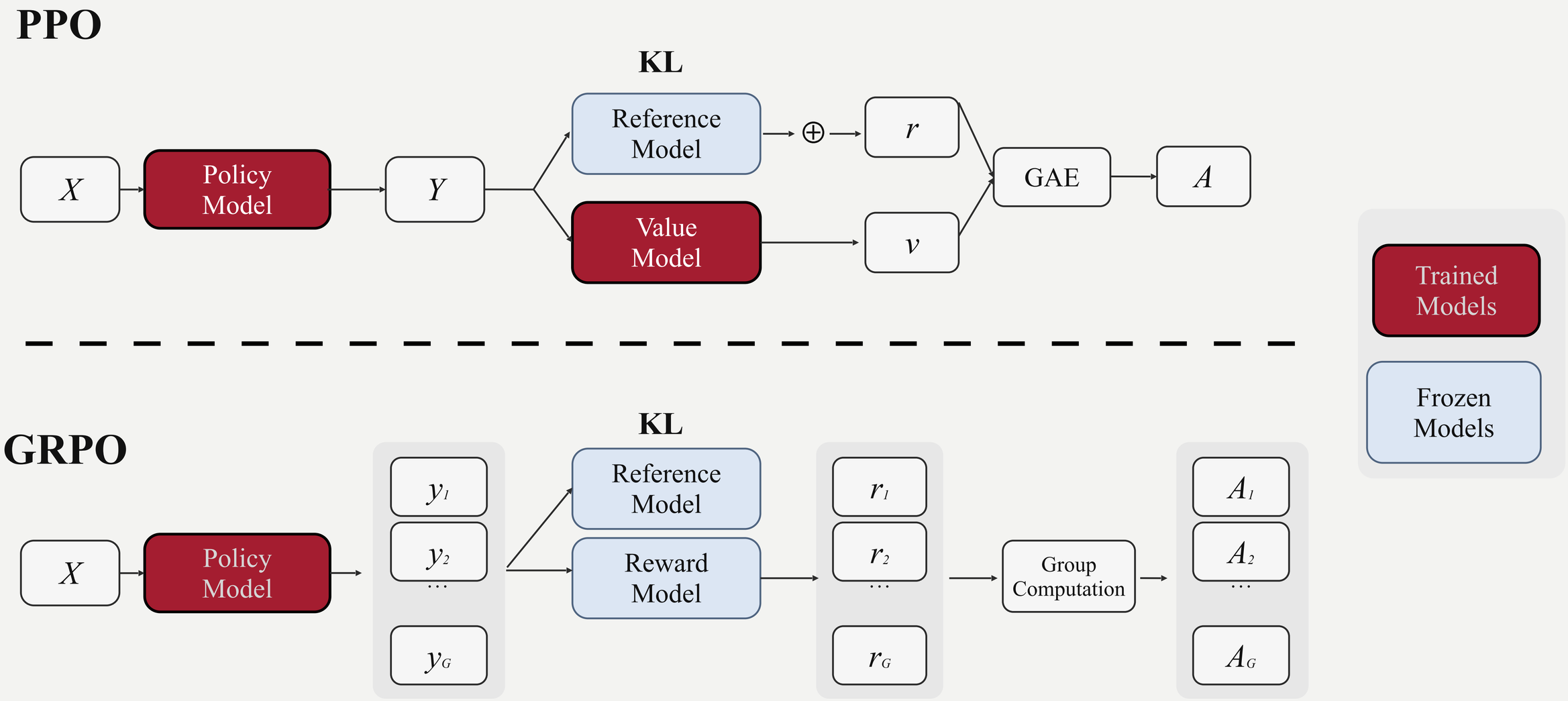
PPO
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GRPO
KL
X
Policy Model
y_1
y_2
y_G
Reference Model
Reward Model
r_1
r_2
r_G
Group Computation
A_1
A_2
A_G

Group Relative Policy Optimization

For each question q, **GRPO samples** a group of outputs $\{o_1, o_2, \cdots, o_G\}$ from the old policy $\pi_{\theta_{\text{old}}}$ and then optimizes the policy model by maximizing the following objective

$$\mathbb{E}_{q\sim P(Q),\,\{o_i\}_{i=1}^{G}\sim\pi_{\theta_{\text{old}}}(O|q)}\left[\frac{1}{G}\sum_{i=1}^{G}\frac{1}{|o_i|}\sum_{t=1}^{|o_i|}\left\{\min\left[\frac{\pi_\theta(o_{i,t}\mid q,o_{i,<t})}{\pi_{\theta_{\text{old}}}(o_{i,t}\mid q,o_{i,<t})}\hat{A}_{i,t},\ \text{clip}\left(\frac{\pi_\theta(o_{i,t}\mid q,o_{i,<t})}{\pi_{\theta_{\text{old}}}(o_{i,t}\mid q,o_{i,<t})},1-\epsilon,1+\epsilon\right)\hat{A}_{i,t}\right]\right\}\right]$$

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where the advantage $A_{i,t}$ is computed using rewards $\mathbf{r} = \{r_1, r_2, \cdots, r_G\}$:

$$\hat{A}_{i,t} = \tilde{r}_i = \frac{r_i - \text{mean}(\mathbf{r})}{\text{std}(\mathbf{r})},$$

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NPG allows one to have big jumps in parameter space, as long as the outcome (distribution) does not change too much

PPO is a more practical versions of NPG — making NPG really scalable while maintaing the high level idea of NPG

GRPO is a more practical versions of PPO — removing the value function